Quantitative Analysis of Watershed Geomorphology

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Abstract—Quantitative geomorphic methods developed within the past few years provide means of measuring size and form properties of drainage basins. Two general classes of descriptive numbers are (1) linear scale measurements, whereby geometrically analogous units of topography can be compared as to size; and (2) dimensionless numbers, usually angles or ratios of length measures, whereby the shapes of analogous units can be compared irrespective of scale.

Linear scale measurements include length of stream channels of given order, drainage density, constant of channel maintenance, basin perimeter, and relief. Surface and cross-sectional areas of basins are length products. If two drainage basins are geometrically similar, all corresponding length dimensions will be in a fixed ratio.

Dimensionless properties include stream order numbers, stream length and bifurcation ratios, junction angles, maximum valley-side slopes, mean slopes of watershed surfaces, channel gradients, relief ratios, and hypsometric curve properties and integrals. If geometrical similarity exists in two drainage basins, all corresponding dimensionless numbers will be identical, even though a vast size difference may exist. Dimensionless properties can be correlated with hydrologic and sediment-yield data stated as mass or volume rates of flow per unit area, independent of total area of watershed.

Introduction—Until about ten years ago the geomorphologist operated almost entirely on a descriptive basis and was primarily concerned with the history of evolution of landforms as geological features. With the impetus given by *Horton* [1945], and under the growing realization that the classical descriptive analysis had very limited value in practical engineering and military applications, a few geomorphologists began to attempt quantification of landform description.

This paper reviews progress that has been made in quantitative landform analysis as it applies to normally developed watersheds in which running water and associated mass gravity movements are the chief agents of form development. The treatment cannot be comprehensive; several lines of study must be omitted. Nevertheless, this paper may suggest what can be done by systematic approach to the problem of objective geometrical analysis of a highly complex surface.

Most of the work cited has been carried out at Columbia University over the past five years under a contract with the Office of Naval Research, Geography Branch, Project NR 389-042 for the study of basic principles of erosional topography. References cited below give detailed explanations of techniques and provide numerous examples taken from field and map study.

Dimensional analysis and geometrical similarity—We have attempted to base a system of quantitative geomorphology on dimensional analysis and principles of scale-model similarity [*Strahler*, 1954a, p. 343; 1957]. Figure 1 illustrates the concept of geometrical similarity, with which we are primarily concerned in topographical description. Basins A and B are assumed to be geometrically similar, differing only in size. The larger may be designated as the prototype, the smaller as the model. All measurements of length between corresponding points in the two basins bear a fixed scale ratio, λ. Thus, if oriented with respect to a common center of similitude, the basin mouths Q′ and Q are located at distances r′ and r, respectively, from C; the ratio of r′ to r is λ. In short, all corresponding length measurements, whether they be of basin perimeter, basin length or width, stream length, or relief (h′ and h in lower profile), are in a fixed ratio, if similarity exists.

All corresponding angles are equal in prototype and model (Fig. 1). This applies to stream junction angles α' and α, and to ground slope angles β' and β. Angles are dimensionless properties; hence the generalization that in two geometrically similar systems all corresponding dimensionless numbers or products describing the geometry must be equal.

Studies of actual drainage basins in differing environments show that in many comparisons in homogeneous rock masses, geometrical similarity is closely approximated when mean values are considered, whereas in other comparisons, where geologic inhomogeneity exists, similarity is def-

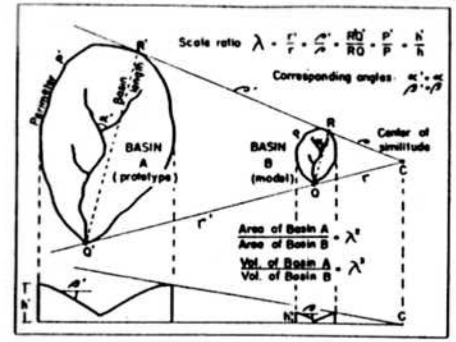

FIG. 1. – Principles of dimensional analysis and geometrical similarity applied to drainage basins

inately lacking [*Strahler*, 1957]. One advantage of using the principles of similarity as a basis of operations is that it focuses attention upon (a) linear scale differences that are independent of form or shape properties, and (b) form differences existing independently of size differences.

The remainder of this paper describes certain landform properties that are dimensionless; others that have dimensions of length or length products and which serve as scale-of-size indicators. Apart from systematizing the analysis, this information is useful in formulation of rational equations relating geomorphic properties to various related or controlling factors with which a significant regression may be expected.

Order analysis—The first step in drainage basin analysis is order designation, following a system only slightly modified from *Horton* [1945, p. 281–282] (Fig. 2). Assuming that the channel-network map includes all intermittent and permanent flow lines located in clearly defined valleys, the smallest finger-tip tributaries are designated Order 1. Where two first-order channels join, a channel segment of Order 2 is formed; where two of Order 2 join, a segment of Order 3 is formed; and so forth. The trunk stream through which all discharge of water and sediment passes is therefore the stream segment of highest order.

Any usefulness which the stream order system may have depends upon the premise that on the average, if a sufficiently large sample is treated, order number is directly proportional to relative watershed dimensions, channel size, and stream discharge at that place in the system. Also, because order number is dimensionless, two drainage basins differing greatly in linear scale can be equated or compared with respect to corresponding

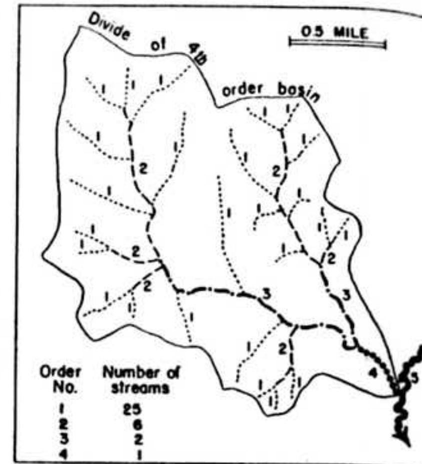

FIG. 2 – Method of designating stream orders (Strahler, 1954a, p. 344)

points in their geometry through use of order number. The first step in drainage-network analysis is the counting of stream segments of each order. This is followed by analysis of the way in which numbers of stream segments change with increasing order.

Bifurcation ratio—*Horton's* [1945, p. 291] law of stream numbers states that the numbers of stream segments of each order form an inverse geometric sequence with order number. This is generally verified by accumulated data [*Strahler*, 1952, p. 1137; *Schumm*, 1956, p. 603] and is conveniently treated as shown in Figure 3. A regression of logarithm of number of streams of each order (ordinate) on stream order (abscissa) generally yields a straight-line plot with very little scatter [*Maxwell*, 1955]. Even though the function relating these variables is defined only for integer values of the independent variable, a regression line is fitted; the slope of the line, or regression coefficient *b* is used. The anti-logarithm of *b* is equivalent to Horton's bifurcation ratio r_b and in this case has the value of 3.52. This means that on the average there are three and one-half times as many streams of one order as of the next higher order.

One might think that the bifurcation ratio would constitute a useful dimensionless number for expressing the form of a drainage system. Actually the number is highly stable and shows a small range of variation from region to region or environment to environment, except where powerful geologic controls dominate. *Coates* [1956, Table 3]

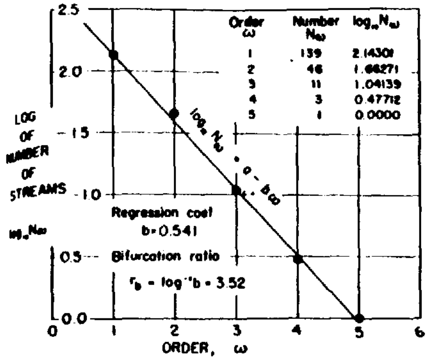

Fig. 3 – Regression of number of stream segments on stream order; data from *Smith* (1953, Plate 8)

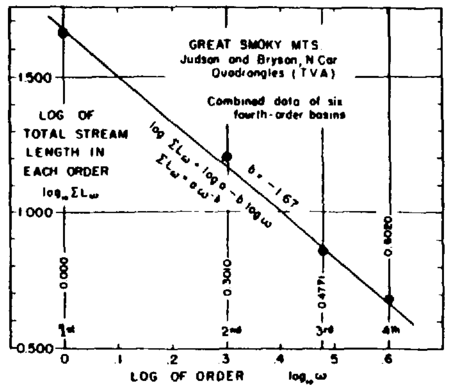

Fig. 4 – Regression of stream length on stream order; stream lengths of six fourth-order basins have been summed for each order to reduce effects of chance variations.

found bifurcation ratios of first-order to second-order streams to range from 4.0 to 5.1; ratios of second-order to third-order streams to range from 2.8 to 4.9. These values differ little from *Strahler's* [1952, p. 1134].

Frequency distribution of stream lengths—Length of stream channel is a dimensional property which can be used to reveal the scale of units comprising the drainage network. One method of length analysis is the measurement of length of each segment of channel of a given stream order. For a given watershed these lengths can be studied by frequency distribution analysis [*Schumm*, 1956, p. 607]. Stream lengths are strongly skewed right, but this may be largely corrected by use of logarithm of length. Arithmetic mean, estimated population variance, and standard deviation serve as standards of description whereby different drainage nets can be compared and their differences tested statistically [*Strahler*, 1954b].

Relation of stream length to stream order—Still another means of evaluating length relationships in a drainage network is to relate stream length to stream order. A regression of logarithm of total stream length for each order on logarithm of order may be plotted (Fig. 4). Again, the function is defined only for integer values of order. Several such plots of length data made to date seem to yield consistently good fits to a straight line, but the general applicability of the function is not yet established, as in the case of the law of stream numbers.

The slope of the regression line *b* (Fig. 4) is the exponent in a power function relating the two variables. Marked differences observed in the exponent suggest that it may prove a useful measure of the changing length of channel segments as order changes. Because this is a nonlinear variation, the assumption is implicit that geometrical similarity is not preserved with increasing order of magnitude of drainage basin.

Drainage basin areas—Area of a given watershed or drainage basin, a property of the square of length, is a prime determinant of total runoff or sediment yield and is normally eliminated as a variable by reduction to unit area, as in annual sediment loss in acre-feet per square mile. In order to compare drainage basin areas in a meaningful way, it is necessary to compare basins of the same order of magnitude. Thus, if we measure the areas of drainage basins of the second order, we are measuring corresponding elements of the systems. If approximate geometrical similarity exists, the area measurements will then be indicators of the size of the landform units, because areas of similar forms are related as the square of the scale ratio.

Basin area increases exponentially with stream order, as stated in a law of areas [*Schumm*, 1956, p. 606], paraphrasing Horton's law of stream lengths.

Schumm [1956, p. 607] has shown histograms of the areas of basins of the first and second orders and of patches of ground surface too small to have channels of their own. Basin area distributions are strongly skewed, but this is largely corrected by use of log of area. Area is measured by planimeter from a topographic map, hence represents projected, rather than true surface area. Estimation of true surface area has been attempted where surface slope is known [*Strahler*, 1956a, p. 579].

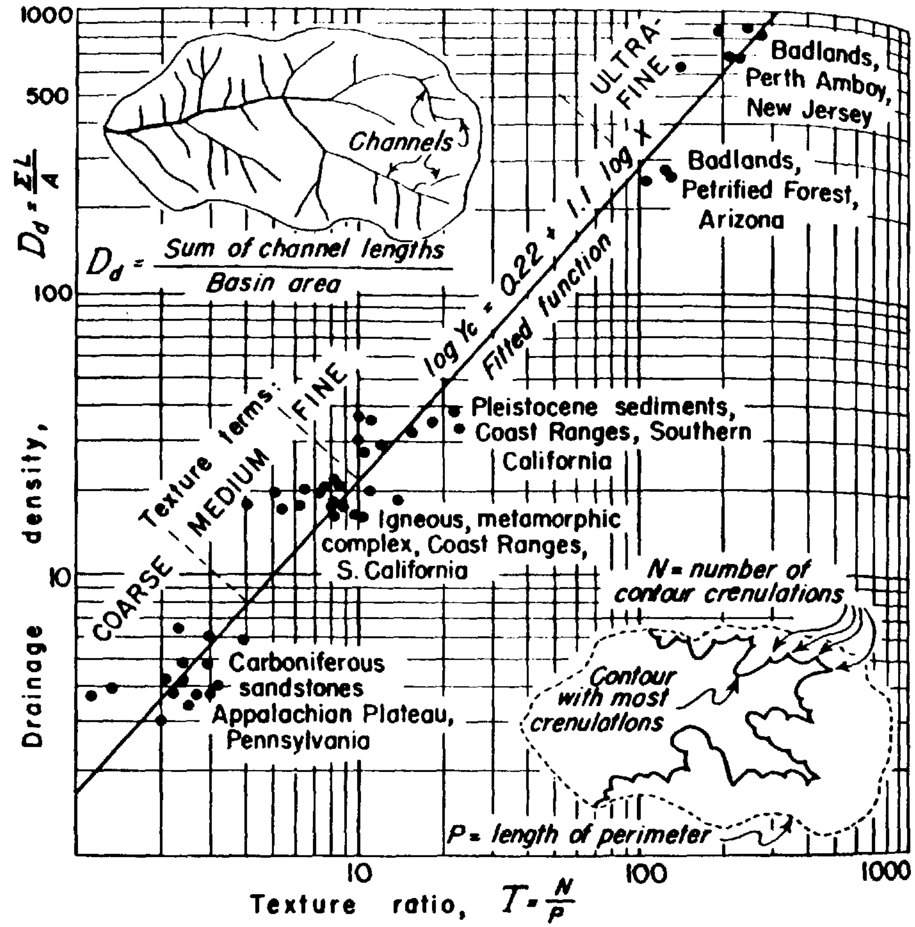

Fig. 5 – Definitions of drainage density and texture ratio (Strahler, 1954a, p. 348)

Drainage density and texture ratio—An important indicator of the linear scale of landform elements in a drainage basin is drainage density, defined by *Horton* [1945, p. 283]. The upper left-hand corner of Figure 5 shows the definition of drainage density as the sum of the channel lengths divided by basin area. Division of length by area thus yields a number with the dimension of inverse of length. In general, then, as the drainage density number increases, the size of individual drainage units, such as the first-order drainage basin, decreases proportionately.

Figure 5 shows the relation between drainage density and a related index, the texture ratio, defined by *Smith* [1950]. Because the contour inflections on a good topographic map indicate the existence of channels too small to be shown by stream symbols, their frequency is a measure of closeness of channel spacing and hence also correlates with drainage density.

Drainage density is scaled logarithmically on the ordinate of Figure 5. The grouped points in the lower left-hand corner of the graph represent basins in resistant, massive sandstones. Here the streams are widely spaced and density is low. The next group of points encountered represents typical densities in deeply weathered igneous and metamorphic rocks of the California coast ranges. In the extreme upper right are points for badlands, where drainage density is from 200 to 900 miles of annels per square mile [*Smith*, 1953; *Schumm*, 56, p. 612].

Because of its wide ratio of variation, drainage nsity is a number of primary importance in dform scale analysis. One might expect that diment yield would show a close positive relation-ip with drainage density. A rational theory of e relation of drainage density to erosion intensity, edicting the morphological changes to be ex-cted when ground surface resistance is lowered y land use, has been outlined by *Strahler* [1956*b*].

Constant of channel maintenance—*Schumm* 956, p. 607] has used the inverse of drainage nsity as a property termed constant of channel aintenance. In Figure 6 the logarithm of basin ea (ordinate) is treated as a function of logarithm total stream channel length (abscissa). Stream ngth is cumulative for a given order and includes l lesser orders; it is thus the total channel length a watershed of given order. Length in this case s projected to the horizontal plane of the map; ue lengths would be obtained by applying a orrection for slope.

An individual plotted point on the graph repre-ents a given stream order in the watershed, as umbered 1 through 5. Using data of the three amples given by Schumm, the sets of points fall ose to a straight line of 45° slope; thus the rela-onship is treated as linear even though plotted ere on log-log paper. If the logarithm of the inter-ept is read at log stream length = 0, and the anti-g of this intercept is taken, we obtain the con-

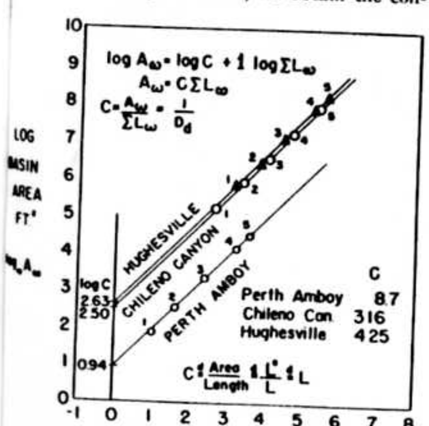

FIG. 6 – Constant of channel maintenance, C. Data replotted on logarithmic scales from *Schumm* (1956, p. 606)

stant of channel maintenance *C* which is actually the slope of a linear regression of area on length.

The value of C = 8.7 in the Perth Amboy bad-lands means that on the average 8.7 sq ft of surface are required to maintain each foot of channel length. In the second example, Chileno Canyon in the California San Gabriel Mountains, 316 sq ft of surface are required to maintain one foot of channel length.

The constant of channel maintenance, with the dimensions of length, is thus a useful means of indicating the relative size of landform units in a drainage basin and has, moreover, a specific genetic connotation.

Maximum valley side slopes—Leaving now the drainage network and what might be classified as planimetric or areal aspects of drainage basins, we turn to slope of the ground surface. This brings into consideration the aspect of relief in drainage basin geometry. One significant indicator of the over-all steepness of slopes in a watershed is the maximum valley-side slope, measured at intervals along the valley walls on the steepest parts of the contour orthogonals running from divides to adjacent stream channels.

Maximum valley-side slope has been sampled by several investigators in a wide variety of geological and climatic environments [*Strahler*, 1950; *Smith*, 1953; *Miller*, 1953; *Schumm*, 1956; *Coates*, 1956; *Melton*, 1957]. Within-area variance is relatively small compared with between-area differences. This slope statistic would therefore seem to be a valuable one which might relate closely to sediment production.

Mean slope curve—Another means of assessing the slope properties of a drainage basin is through the mean slope curve [*Strahler*, 1952, p. 1125–1128]. This requires the use of a good contour topographic map. The problem is to estimate the average, or mean slope of the belt of ground surface lying between successive contours. This may be done by measuring the area of each contour belt with a planimeter and dividing this area by the length of the contour belt to yield a mean width. The mean slope will then be that angle whose tangent is the contour interval divided by the mean belt width. Mean slope of each contour interval is plotted from summit point to basin mouth. Curves of this type will differ from region to region, depending upon geologic structure and the stage of development of the drainage system. If the mean slope for each contour belt is weighted for per cent of total basin surface area, it is possible

to arrive at a mean slope value for the surface of the watershed as a whole.

Slope maps—Another means of determining slope conditions over an entire ground surface of a watershed is through the slope map [*Strahler*, 1956a]. (1) A good topographic map is taken. (2) On this map the slope of a short segment of line normal to the trend of the contours is determined at a large number of points. These may be recorded as tangents or sines, depending upon the kind of map desired. (3) These readings are contoured with lines of equal slope, here called isotangents. (4) The areas between successive isotangents are measured with a planimeter and the areas summed for each slope class. (5) This yields a slope frequency percentage distribution. Because the entire ground surface has been analyzed, the mean, standard deviation, and variance are treated as population parameters, at least for purposes of comparison with small samples taken at random from the same area.

Lines of equal sine of slope, or isosines, may also be drawn. The interval between isosines on the map becomes the statistical class on the histogram. Sine values are designated as *g* values because the sine of slope represents that proportion of the acceleration of gravity acting in a down-slope direction parallel with the ground surface.

Rapid slope sampling—The construction of slope maps and their areal measurement is extremely time-consuming. Experiments have shown that essentially the same information can be achieved by random point sampling [*Strahler*, 1956a, p. 589–595]. Both random coordinate-sampling and grid sampling have been tried. In the random-coordinate method a sample square is scaled in 100 length units per side. From a table of random numbers the coordinates of sample points are drawn for whatever sample size is desired. The grid method does much the same thing, but is not flexible as to sample size.

Point samples, which are easy to take, were compared with the frequency distribution measured from a slope map. Noteworthy is the extremely close agreement in means and variances, and even in the form of the frequency distributions, including a marked skewness. Tests of sample variance and mean are discussed by *Strahler* [1956a].

Chapman [1952] has developed a method of analyzing both azimuth and angle of slope from contour topographic maps. Although based on petrofabric methods and designed largely for use in geological analysis of terrain, the method might be applied to a watershed as a means of assessing both slope steepness and orientation simultaneously.

Relief ratio—*Schumm* [1956, p. 612] has devised and applied a simple statistic, the relief ratio, defined as the ratio between total basin relief (that is, difference in elevation of basin mouth and summit) and basin length, measured as the longest dimension of the drainage basin. In a general way, the relief ratio indicates overall slope of the watershed surface. It is a dimensionless number, readily correlated with other measures that do not depend on total drainage basin dimensions. Relief ratio is simple to compute and can often be obtained where detailed information on topography is lacking.

Schumm [1954] has plotted mean annual sediment loss in acre feet per square mile as a function of the relief ratio for a variety of small drainage basins in the Colorado Plateau province [Fig. 7]. The significant regression with small scatter suggests that relief ratio may prove useful in estimating sediment yield if the parameters for a given climatic province are once established.

Hypsometric analysis—Hypsometric analysis, or the relation of horizontal cross-sectional drainage basin area to elevation, was developed in its modern dimensionless form by *Langbein* and others [1947]. Whereas he applied it to rather large watersheds, it has since been applied to small drainage basins of low order to determine how the mass is

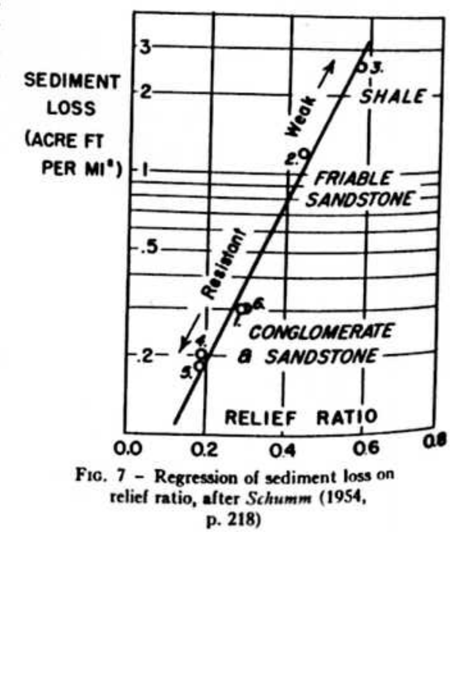

FIG. 7 – Regression of sediment loss on relief ratio, after *Schumm* (1954, p. 218)

distributed within a basin from base to top [*Strahler*, 1952; *Miller*, 1953; *Schumm*, 1956; *Coates*, 1956].

Figure 8 illustrates the definition of the two dimensionless variables involved. Taking the drainage basin to be bounded by vertical sides and a horizontal base plane passing through the mouth, the relative height is the ratio of height of a given contour h to total basin height H. Relative area is the ratio of horizontal cross-sectional area a to entire basin area A. The percentage hypsometric curve is a plot of the continuous function relating relative height y to relative area x.

As the lower right-hand diagram of Figure 8 shows, the shape of the hypsometric curve varies in early geologic stages of development of the drainage basin, but once having attained an equilibrium, or mature stage (middle curve on graph), tends to vary little thereafter. Several dimensionless attributes of the hypsometric curve are measurable and can be used for comparative purposes. These include the integral, or relative

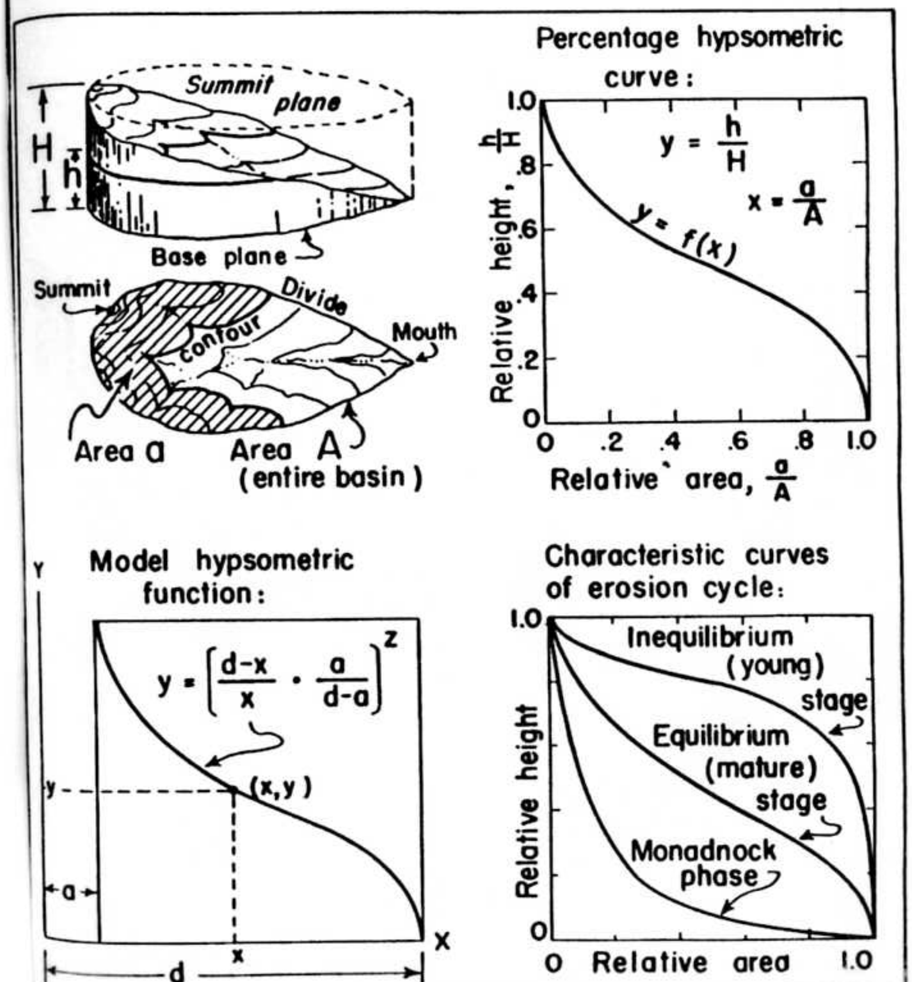

FIG. 8 – Method of hypsometric analysis (Strahler, 1954a. p. 353)

area lying below the curve, the slope of the curve at its inflection point, and the degree of sinuosity of the curve. Many hypsometric curves seem to be closely fitted by the model function shown in the lower left corner of Figure 8, although no rational or mechanical basis is known for the function.

Now that the hypsometric curves have been plotted for hundreds of small basins in a wide variety of regions and conditions, it is possible to observe the extent to which variation occurs. Generally the curve properties tend to be stable in homogeneous rock masses and to adhere generally to the same curve family for a given geologic and climatic combination.

Conclusion—This paper has reviewed briefly a variety of geometrical properties, some of length dimension or its products, others dimensionless, which may be applied to the systematic description of drainage basins developed by normal processes of water erosion. Among the morphological aspects not mentioned are stream profiles and the geometry of stream channels. These, too, are subject to orderly treatment along the lines suggested. The examples of quantitative methods presented above are intended to show that, complex as a landscape may be, it is amenable to quantitative statement if systematically broken down into component form elements. Just which of these measurements or indices will prove most useful in explaining variance in hydrological properties of a watershed and in the rates of erosion and sediment production remains to be seen when they are introduced into multivariate analysis. Already there are definite indications of the usefulness of certain of the measures and it is only a matter of continuing the development of analytical methods until the most important geomorphic variables are isolated.

REFERENCES

CHAPMAN, C. A., A new quantitative method of topographic analysis, *Amer. J. Sci.*, **250**, 428–452, 1952.

COATES, D. R., *Quantitative geomorphology of small drainage basins in southern Indiana*, Of. Nav. Res. Proj. NR 389-042, Tech. Rep. 10 (Columbia Univ. Ph.D. dissertation), 57 pp., 1956.

HORTON, R. E., Erosional development of streams and their drainage basins; hydrophysical approach to quantitative morphology, *Bul. Geol. Soc. Amer.*, **56**, 275–370, 1945.

LANGBEIN, W. B., AND OTHERS, Topographic characteristics of drainage basins, *U. S. Geol. Surv. Water-Supply Paper 968-C*, 157 pp., 1947.

MAXWELL, J. C., The bifurcation ratio in Horton's law of stream numbers, (abstract), *Trans. Amer. Geophys. Union*, **36**, 520, 1955.

MELTON, M. A., *An analysis of the relations among elements of climate, surface properties, and geomorphology*, Of. Nav. Res. Proj. NR 389-042, Tech. Rep. 11 (Columbia Univ. Ph.D. dissertation), 102 pp., 1957.

MILLER, V. C., *A quantitative geomorphic study of drainage basin characteristics in the Clinch Mountain area, Virginia and Tennessee*, Of. Nav. Res. Proj. NR 389-042, Tech. Rep. 3 (Columbia Univ. Ph.D. dissertation), 30 pp., 1953.

SCHUMM, S. A., The relation of drainage basin relief to sediment loss, *Pub. International Association of Hydrology*, IUGG, Tenth Gen. Assembly, Rome, 1954, **1**, 216–219, 1954.

SCHUMM, S. A., Evolution of drainage systems and slopes in badlands at Perth Amboy, New Jersey, *Bul. Geol. Soc. Amer.*, **67**, 597–646, 1956.

SMITH, K. G., Standards for grading texture of erosional topography, *Amer. J. Sci.*, **248**, 655–668, 1950.

SMITH, K. G., *Erosional processes and landforms in Badlands National Monument, South Dakota*, Of. Nav. Res. Proj. NR 389-042, Tech. Rep. 4 (Columbia Univ. Ph.D. dissertation), 128 pp., 1953.

STRAHLER, A. N., Equilibrium theory of erosional slopes approached by frequency distribution analysis, *Amer. J. Sci.*, **248**, 673–696, 800–814, 1950.

STRAHLER, A. N., Hypsometric (area-altitude) analysis of erosional topography, *Bul. Geol. Soc. Amer.*, **63**, 1117–1142, 1952.

STRAHLER, A. N., Quantitative geomorphology of erosional landscapes, *C.-R. 19th Intern. Geol. Cong.*, Algiers, 1952, sec. 13, pt. 3, pp. 341–354, 1954a.

STRAHLER, A. N., Statistical analysis in geomorphic research, *J. Geol.*, **62**, 1–25, 1954b.

STRAHLER, A. N., Quantitative slope analysis, *Bul. Geol. Soc. Amer.*, **67**, 571–596, 1956a.

STRAHLER, A. N., The nature of induced erosion and aggradation, pp. 621–638, Wenner-Gren Symposium Volume, *Man's role in changing the face of the Earth*, Univ. Chicago Press, Chicago, Ill., 1193 pp., 1956b.

STRAHLER, A. N., *Dimensional analysis in geomorphology*, Of. Nav. Res. Proj. NR 389-042, Tech. Rep. 7, Dept. Geol., Columbia Univ., N. Y., 43 pp., 1957.

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